REACTIVE PATH RELINKING ALGORITHM

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OUTLINE

1. PROBLEM DESCRIPTION
2. REACTIVE PATH RELINKING ALGORITHM
3. COMPUTATIONAL RESULTS
4. CONCLUSIONS AND FUTURE RESEARCH

1.PROBLEM DEFINITION

FACILITY LOCATION PROBLEM

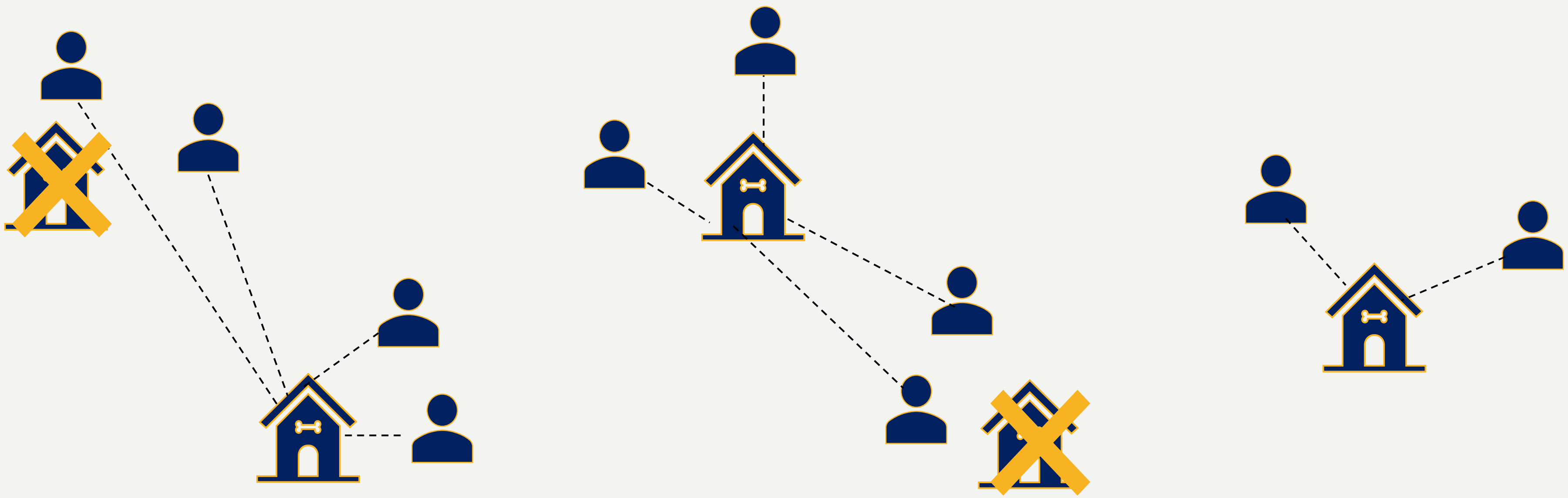
They consist of finding the **BEST** location to site a set of facilities which must serve a set of demand points.

Which are the best locations?

Min. total distance
Min. maximum distance
Max. coverage
Balance the distance
Balance workload
…

FACILITY LOCATION PROBLEM

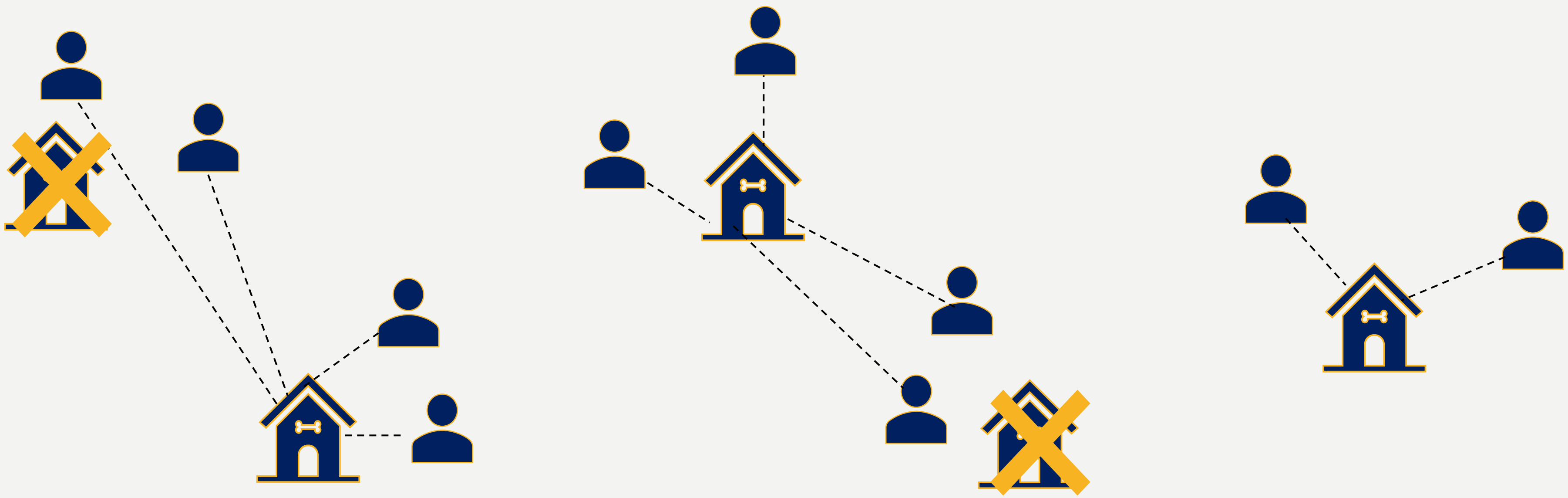
$N = \{1, \dots, n\}$ set of demand points
p $\subset M = \{1, \dots, m\}$ set of candidate locations to host p facilities.



d_{ij} : distance between the demand point i and the facility j.
It is assumed that all demand points are serviced/assigned to their closest facility.

THE BI-OBJECTIVE P-MEDIAN P-DISPERSION PROBLEM (BpMD)

1. Minimize the maximum distance between each demand point and its closest selected facility (pMP)
2. Maximize the distance between selected facilities (pDP)



THE BI-OBJECTIVE P-MEDIAN P-DISPERSION PROBLEM (BpMD)

$$f_{pM}(S) \leftarrow \sum_{j \in N \setminus S} \min_{i \in S} d_{ij}$$

Then, the objective of the p-Median problem is to find a solution $S^\star_{pM}$ with the minimum f_{pM} value. More formally,

$$S^\star_{pM} \leftarrow \arg\min_{S \in \mathbb{P}} f_{pM}(S)$$

$$f_{pD}(S) \leftarrow \min_{i,j \in S, i \neq j} d_{ij}$$

The p-Dispersion problem then consists in finding a solution $S^\star_{pD}$ with the maximum f_{pD} value. Specifically,

$$S^\star_{pD} \leftarrow \arg\max_{S \in \mathbb{P}} f_{pD}(S)$$

BpMD problem

THE BI-OBJECTIVE P-MEDIAN P-DISPERSION PROBLEM (BpMD)

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	Facilities	f_{pM}	f_{pD}
1	(1,1);(1,4);(2,2)	3.83	1.41
2	(1,1);(1,4);(3,2)	3.24	2.24
3	(1,1);(1,4);(4,4)	3.65	**3.00**
4	(1,1);(2,2);(3,2)	4.47	1.00
5	(1,1);(2,2);(4,4)	3.24	1.41
6	(1,1);(3,2);(4,4)	3.83	2.24
7	(1,4);(2,2);(3,2)	3.65	1.00
8	(1,4);(2,2);(4,4)	**2.41**	2.24
9	(1,4);(3,2);(4,4)	3.24	2.24
10	(2,2);(3,2);(4,4)	3.65	1.00

Table 1: All feasible solutions.

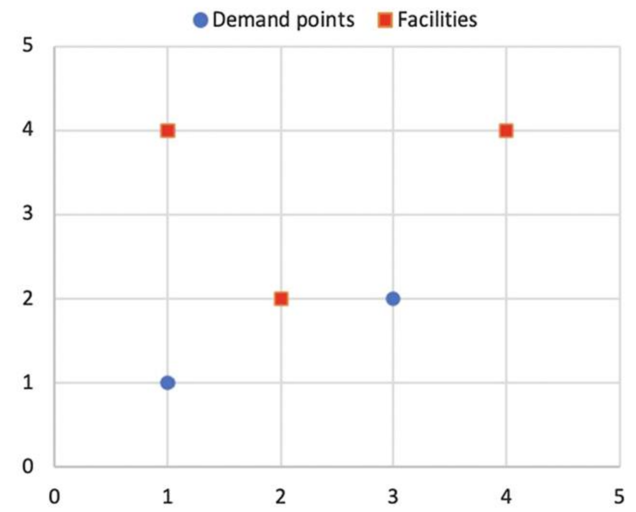


Fig. 1 $(f^{\star}_{pM}, f_{pD}) = (2.41, 2.24)$

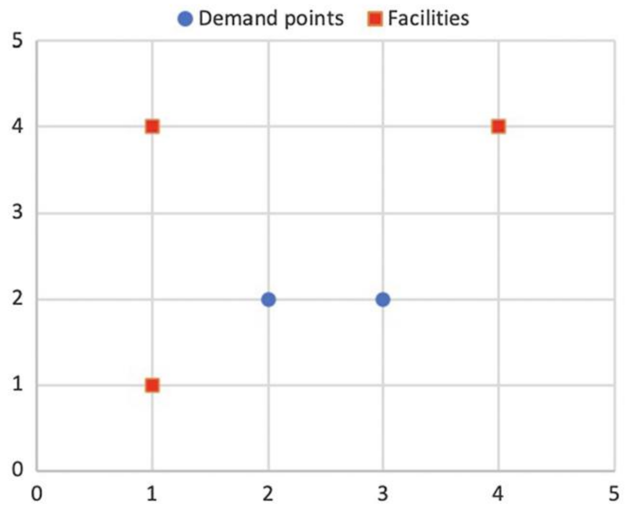


Fig. 2 $(f_{pM}, f^{\star}_{pD}) = (3.65, 3.00)$

THE BI-OBJECTIVE P-MEDIAN P-DISPERSION PROBLEM (BpMD)

Minimization bi-objective combinatiorial optimization problem:

$$\mathrm{Min}_{S \in \Omega} F(S) = \left[f_{pM}(S), -f_{pD}(S)\right]^T$$

TASK: To calculate a set of Pareto-optimal solutions, also known as non-dominated or efficient solutions, to get the **Pareto front**.

Given two solutions S1 and S2 it is said that:

- S_1 **weakly dominates** S_2, denoted as $S_1 \preccurlyeq S_2$ if and only if $F(S_1) \leq F(S_2)$.
- S_1 **dominates** S_2, denoted as $S_1 \prec S_2$ if and only if $S_1 \preccurlyeq S_2$ and at least one objective function of $F(S_1)$ is strictly better than the corresponding one of $F(S_2)$.
- S_1 **strictly dominates** S_2, denoted as $S_1 \preccurlyeq\preccurlyeq S_2$ if and only if $F(S_1) < F(S_2)$.

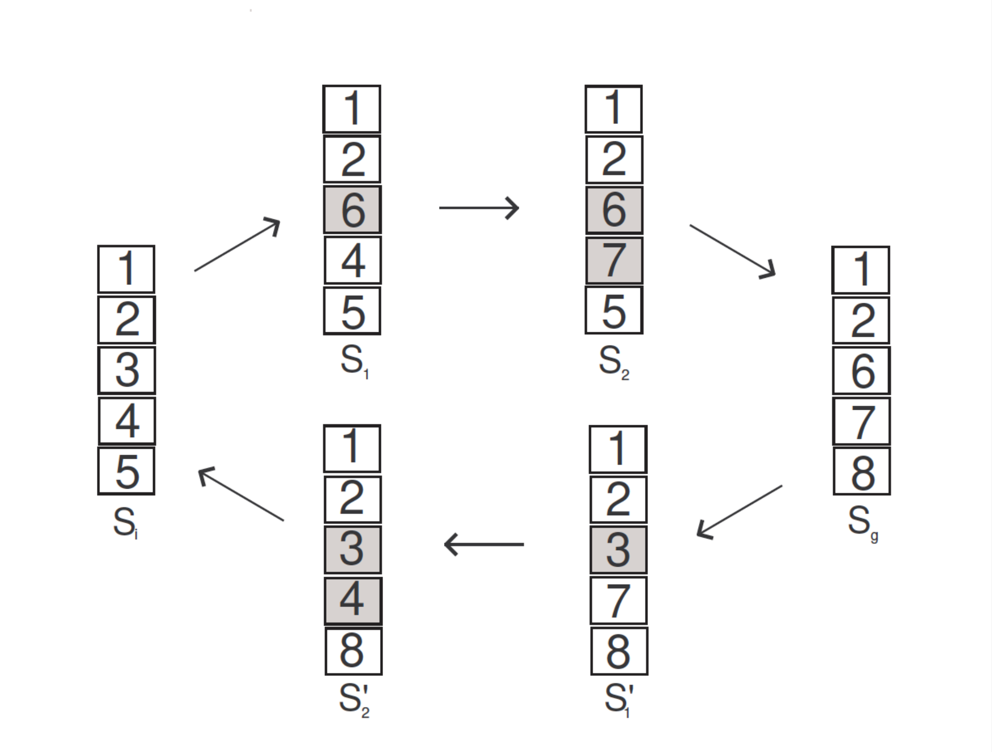
2. REACTIVE PATH RELINKING ALGORITHM

PR ALGORITHM

PATH RELINKING (PR) generates new intermediate solutions that would be hopefully better than these pairs of connected solutions.

IPR ALGORITHM

INTERIOR PATH RELINKING (IPR)

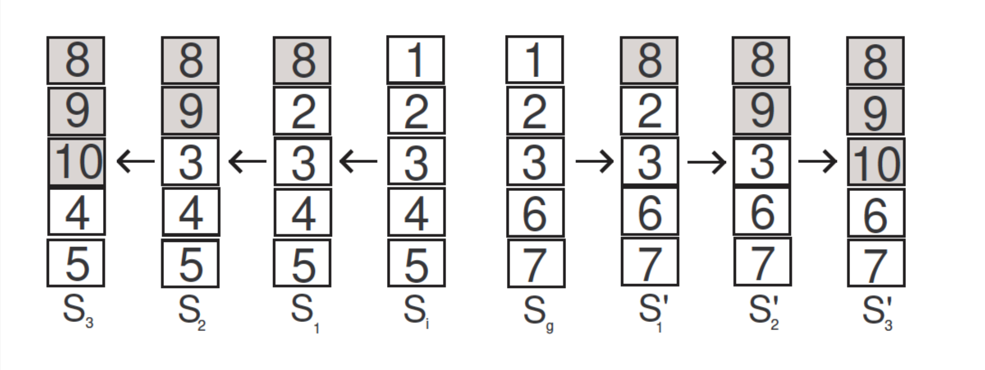


Resende M, Martí R, Gallego M, Duarte A (2010) Grasp and path relinking for the max-min diversity problem. Comput Oper Res 37:498–508

INCLUDE ELEMENTS TO BECOME SOLUTIONS MORE SIMILAR

EPR ALGORITHM

EXTERIOR PATH RELINKING (EPR)



INCLUDE ELEMENTS
TO BECOME SOLUTIONS
MORE DIFFERENT

Duarte A, Sánchez-Oro J, Resende MG, Glover F, Martí R (2015) Greedy randomized adaptive search procedure with exterior path relinking for differential dispersion minimization. Inf Sci 296:46–60

RPR ALGORITHM

REACTIVE PATH RELINKING (PR) combines IPR and EPR by analyzing the similarity between the initiating (S_i) and the guiding (S_g) solutions.

$$\text{Similarity} = |S_i \cap S_g|$$

Threshold:

$$k \times p$$

- If S_i and S_g are different IPR (**INTENSIFICATION**)
- If S_i and S_g are similar EPR(**DIVERSIFICATION**)

RPR ALGORITHM

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* Construction phase based on a greedy algorithm to obtain the Pareto Front

Step 1. A value β is fixed with $\beta = \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$

Step 2. Facility, v, is selected with v=1,…,n

Step 3. The remaining facilities are included in the solution S according to a greedy function:

$$\beta \cdot f_{pM}(S' \cup \{v\}) - (1-\beta) \cdot f_{pD}(S' \cup \{v\}).$$

Step 4. Repeat for each β value and starting from each vertex v .

*Improvement phase

$$\mathcal{N}(S) \leftarrow \{(S \setminus \{u\}) \cup \{v\} : u \in S, v \in N \setminus S\}$$

3. COMPUTATIONAL RESULTS

COMPUTATIONAL RESULTS

40 pmed instances: number of demand points and candidate facilities are in the range of 100 to 900, while the number of selected facilities is in the range of 5 to 200.

19 medium-sized instances: with n = 250 and p = 25, named as D250 and 10 instances with n = 350 and p = 35, named D350.

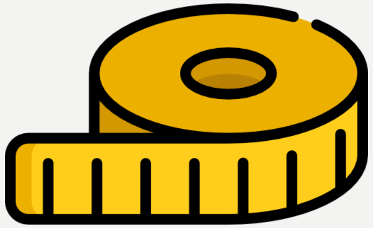
15 kmedian large-sized instances: the smallest one contains n = 1000 nodes, then, the next instances increment by 1000 nodes until reach n = 5000, each instance has 3 different p, 0.05n, 0.15n and 0.25n.

Intel Core i7-7700HQ (4x 2.8 GHz) with 8GB RAM, and the algorithms were implemented using Java 9.

COMPUTATIONAL RESULTS

Comparison:

MOEA/D
NSGA-II
SPEA2
Scatter Search
ϵ-Constraint method
RPR



Metrics:

The number of efficient solutions in the Pareto front (|A|)
The coverage metric, C(A, B)
The spread (Δ)
The hypervolume (HV)
The ϵ-indicator (ϵ)
The generational distance (GD)
The inverted generational distance (IGD)
The computing time (CPU)

COMPUTATIONAL RESULTS

40 pmed instances (small instances)

Algorithm	$\|A\|$	$C(A, B)$	Δ	HV	ϵ	GD	IGD	CPU
MOEA/D	8.45	0.96	**1.05**	0.25	0.69	2787.76	5952.33	3600.00
NSGA-II	20.90	0.95	0.98	0.33	0.58	2028.01	5965.03	3600.00
SPEA2	20.20	0.95	0.98	0.31	0.61	2124.67	5963.36	3600.00
SS	7.43	0.97	0.99	0.18	0.82	2713.59	6086.41	**750.00**
ϵ-C	17.45	0.75	0.94	0.52	0.22	1951.12	5868.96	3579.70
RPR	**31.90**	**0.13**	0.96	**0.70**	**0.03**	**1409.76**	**5564.13**	1644.12

COMPUTATIONAL RESULTS

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19 medium-sized instances

Algorithm	$\|A\|$	$C(A, B)$	Δ	HV	ϵ	GD	IGD	CPU
MOEA/D	10.55	0.95	**1.01**	0.42	0.52	448.90	1384.95	3600.00
NSGA-II	19.25	0.95	0.99	0.46	0.48	336.41	1392.45	3600.00
SPEA2	21.95	0.97	0.99	0.51	0.41	311.05	1395.45	3600.00
SS	8.85	0.91	0.98	0.45	0.33	540.40	1515.27	1220.36
ϵ-C	7.95	**0.00**	0.98	**0.75**	**0.06**	518.52	**1370.15**	1516.61
RPR	**44.40**	0.67	0.98	0.72	0.08	**229.38**	1380.25	**391.57**

COMPUTATIONAL RESULTS

15 k-median instances (large instances)

Algorithm	$\|A\|$	$C(A, B)$	Δ	HV	ϵ	GD	IGD	CPU
MOEA/D	2.67	0.38	**1.00**	0.13	2.12	334006.89	**520357.18**	**3600.00**
NSGA-II	2.87	0.61	**1.00**	0.18	2.34	362286.49	527342.44	**3600.00**
SPEA2	3.67	0.60	**1.00**	**0.21**	2.26	334101.08	528861.44	**3600.00**
SS	1.53	0.67	0.93	0.03	0.89	453844.16	542939.49	**3600.00**
RPR	**5.93**	**0.16**	**1.00**	0.11	**0.87**	**245574.20**	562329.71	**3600.00**

4.CONCLUSIONS AND FUTURE RESEARCH

CONCLUSIONS

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- **The proposal outperforms not only the state-of-the-art algorithm but also the most competitive evolutionary algorithms proving the superiority of our algorithm.**
- **This variant of PR is able to decide with strategy is better when combining solutions. RPR allows to intensify or diversify depending on the selected solutions.**

Lozano-Osorio, I.; Sánchez-Oro, J.; López-Sánchez, A.D..; Duarte, A. A reactive path relinking algorithm for solving the bi-objective p-Median and p-Dispersion problem. Soft Computing **2023**, 27: 8029-8059. https://doi.org/10.1007/s00500-023-07994-4

FUTURE RESEARCH

- To solve a variety of combinatorial problems to prove the powerful of the RPR.
- To calibrate the threshold of the similarity depending on the considered problem by using maching learning strategies to detect if it is needed to intensify or diversity, which allows the algorithm to adapt the threshold at any moment when applying the algorithm.

THANK YOU FOR YOUR ATTENTION