Path Relinking for high dimensional continuous optimization

Francisco Gortázar[1], Rafael Martí[2], Abraham Duarte[1]

[1]DCC, Universidad Rey Juan Carlos
Tulipán, s/n – CP28933 Madrid, Spain
abraham.duarte@urjc.es, francisco.gortazar@urjc.es

[2]Universitat de València
Dr. Moliner 50, CP 46100 Valencia, Spain
rafael.marti@uv.es

Abstract

In this paper we present an evolutionary Path Relinking algorithm for solving unconstrained high dimensional optimization problems. We target 19 types of global optimization functions, for which we test how the optimization strategies that we are proposing perform with increasing dimension. We include in our computational comparison three algorithms from the state-of-the-art, and show that our algorithm is competitive with them in high dimensional problems.

1. Introduction

Path Relinking, PR, is an optimization methodology that explores trajectories connecting elite solutions identified in previous search efforts [3]. PR generates a path from an initiating solution (in the set of elite solutions), and incrementally changes attributes of this solution with those of a guiding solution. Resende and Werneck [10] introduced Evolutionary Path Relinking (EvPR) as a post-processing phase for GRASP with PR in which the solutions in the elite set evolve in a similar way as in other evolutionary algorithms such as scatter search [6] or genetic algorithms [9].

In this paper we propose an adaptation of the Evolutionary Path Relinking methodology for continuous optimization. Specifically, we address the unconstrained nonlinear optimization problem in the form:

$$Minimize\ f(x):$$

$$L \leq x \leq U \qquad (1)$$

where $x = (x_1, \dots, x_n)$ is a vector of continuous variables with lower bound L and upper bound U. This problem has been the subject of intensive study in recent years. In particular, there are three evolutionary algorithms considered to be the most effective for this type of problem: Differential Evolution (DE) [12], a parallel direct search method, Real-coded CHC [2], a genetic algorithm with several components that induce a strong diversity, and STS [1], an adaptive memory programming method integrating Scatter Search and Tabu Search methodologies. These three methods have shown to be competitive in high dimensions. In this paper we test our EvPR method on a standard benchmark of 19 global optimization functions [8] with 5 different dimension sizes totalizing 95 instances and compare our results with those of DE, STS and Real-coded CHC.

2. Evolutionary Path Relinking

Our EvPR algorithm starts with the generation of a set D with $DSize$ initial solutions using techniques from the area of statistics known as Design of Experiments. Specifically, D is constructed using the fractional factorial design proposed in [11] (i.e., Taguchi's arrays) as a way of generating diverse solutions [7]. Thus, D contains solutions uniformly distributed in the solution space. We build the initial elite set ES with the best b solutions in D(in terms of quality because D is diverse by construction).

Following the standard design of EvPR, once ES has been populated, we apply Path Relinking to the solutions in ES. The solutions obtained are candidates to enter ES, and PR is again applied to them as long as new solutions enter ES. This way we say that ES evolves. Specifically, we first generate $NewSubsets$ consisting of sets (a, x, y) of three solutions in the elite set. We select one-by-one sets from $NewSubsets$ and perform the relinking method on each of them. The best solution obtained by the relinking process is improved, added to a $Pool$ of relinked solutions and the corresponding set (a, x, y) is removed from $NewSubsets$. When we have considered all sets in $NewSubsets$, we test whether solutions in $Pool$ can enter the elite set or not (according to both quality and diversity). If at least one new solution qualifies to enter to the elite set, we start a new global iteration, otherwise we rebuild the elite set.

When no new solution qualify to enter the ES, we apply an Elite Set rebuilding, which consists of continuing exploring solutions in D that remain unexplored. In particular, we select the next solution in D and directly submit it to the relinking process, which is applied between this solution and two elite solutions x and y chosen with a probability according to their quality. The solution that results from the relinking method is improved and tested for inclusion in elite set. This process is repeated b times. The new solutions inserted in the elite set replace those solutions that are more similar to them, in order to maintain diversity within the elite set, and they can be used in further applications of PR.

2.1 Improvement method

The improvement method performs line searches coupled with the simplex method and it is applied to the best solution from the set of original solutions and to the best solution in the Path Relinking method below. As it is usual in global optimization, the search space is discretized using a grid of size h. In order to reduce the search space we selectively apply line searches to the $n/2$ most promising variables. This is done by computing, for each variable $i = \{1, \dots, n\}$, two points: $x + he_i$ and $x - he_i$, where e_i is a vector unit for variable i. Then, we order the variables according to these values, being the variable i with a better value the first one, and so on.

Once a variable i has been selected, a line search over that variable is performed. This line search consists of discretizing the solution space and explore those points with the form $x + khe_i$, with k in the integer interval $[-20, \dots, 20]$, i.e. we explore at most 40 points for each variable. It is possible to explore less than 40 points, because these points must hold the constraint $l \leq x + khe_i \leq u$. Again, in order to reduce the number of evaluations, we consider a *first improving* strategy, where we randomly scan these points and select the first one that improves the current solution. When we finish with a variable, we resort to the next one, until the first $n/2$ variables have been considered. Then, we recalculate the contribution of each variable and perform another $n/2$ line searches. This process stops when no further improvement is possible or when a maximum of 10 iterations is reached.

2.2 Linking solutions

We have designed a path relinking method that explores the path from an initiating solution a to a pair of guiding solutions x and y. Giving a set of solutions(a, x, y), the relinking consists of moving first from a in the direction given by the vector from a to x. Specifically, we evaluate the solutions in the form:

$$a(1) = a + \frac{1}{k}(x - a), \dots, a(k-1) = a + \frac{1}{2}(x - a) \tag{2}$$

where $a(j)$ is the best solution found in the path from a to x, we then scan the solutions in the vector from $a(j)$ to y selecting the best overall one:

$$a(k) = a(j) + \frac{1}{k}\big(y - a(j)\big), \dots, a(2k-2) = a(j) + \frac{1}{2}(y - a(j)) \tag{3}$$

As mentioned above, we apply the improvement to the best solution found so far.

3. Experimental results

We have tested the efficiency of our EvPR procedure by applying it to solve 19 functions for 5 different dimensions [4]. The optimum of these 95 instances is known by construction. We report for each instance the gap with respect to the optimum, defined as $f(x) - f(op)$, where x is a solution and op is the optimum of each instance. We run each algorithm 25 times for each function for a maximum of $5000n$ evaluations, where $n = \{50,100,200,500,1000\}$ is the problem dimension. We compare our method with three state-of-the-art methods: STS [1], DE [12], and CHC [2]. Results in Table 1 show that both DE and EvPR are the best methods overall (with no clear winner). It seems that EvPR provides better results as the dimensionality increases. On the contrary, DE seems better suited for lower dimensions.

	DE	**CHC**	**STS**	**EvPR**
50	1.74E+001	1.76E+005	6.92E+001	2.41E+001
100	5.44E+001	3.70E+005	5.02E+002	1.62E+002
200	3.64E+002	1.37E+006	2.26E+003	7.45E+002
500	3.39E+003	1.80E+006	1.38E+004	3.82E+003
1000	1.33E+004	4.60E+006	6.27E+004	1.16E+004
Average	3.43E+03	1.67E+06	1.59E+04	3.27E+03

Table 1: Average gap over 25 runs

4. Conclusions

We have described an evolutionary path relinking algorithm for high dimensional continuous optimization problems. We have tested this method by performing an empirical comparison with previous methods, and the results show that our EvPR is a competitive for high dimensional continuous nonlinear functions.

References

[1] A.Duarte, R. Martí, F. Glover, and F. Gortazar. Hybrid Scatter Tabu Search for Unconstrained Global Optimization. *Annals of Operations Research*. 183(1), 95-123, 2010.

[2] L.J. Eshelman, and J. D. Schaffer. Real-coded genetic algorithm and interval schemata. Foundation of Genetic Algorithms. 187–202, 1993.

[3] F.Glover, M. Laguna. Tabu search. Kluwer, Boston, 1997.

[4] F. Herrera, M. Lozano and D. Molina. Test Suite for the Special Issue of Soft Computing on Scalability of Evolutionary Algorithms and other Metaheuristics for Large Scale Continuous Optimization Problems. Technical report. University of Granada, February 9, 2010.

[5] M. Laguna and R. Martí. GRASP and Path Relinking for 2-Layer Straight Line Crossing Minimization. *INFORMS Journal on Computing*. 11(1), 44-52, 1999.

[6] M. Laguna, and R. Martí. *Scatter search—methodology and implementations in C*. Boston: Kluwer Academic, 2003.

[7] M. Laguna and R. Martí. Experimental Testing of Advanced Scatter Search Designs for Global Optimization of Multimodal Functions. *Journal of Global Optimization*. 33, 235-255, 2005.

[8] M. Lozano, F. Herrera, D. Molina (Eds.). Scalability of Evolutionary Algorithms and other Metaheuristics for Large Scale Continuous Optimization Problems. *Soft Computing*. 2011.

[9] Z. Michalewicz, D. B. Fogel. How to Solve It: Modern Heuristics. Springer, 2004.

[10] M. G.C. Resende and R. F. Werneck. A hybrid heuristic for the p-median problem. *Journal of Heuristics*. 10, 59–88, 2004.

[11] R.Roy. A Primer on the Taguchi Method. Van Nostrand Reinhold. New York, 1990.

[12] R.Storn and K. Price. Differential evolution - a simple and efficient heuristic for global optimization over continuous spaces. *Journal of Global Optimization*. 11, 341–359, 1997.